Evaluation of the Efficiency of Stimulus Measures

Rafael Yanushevsky, Camilla Yanushevsky

ABSTRACT

The impact of expansionary fiscal policy intended to increase economic growth by using stimulus packages is analyzed by considering the debt to GDP ratio dynamics model. It is shown that for the data characterizing the current state of the U.S. economy the government investment in infrastructure and tax cuts alone cannot decrease the debt to GDP ratio. The paper contributes to the on-going fiscal policy debate whether government investment in infrastructure and tax cuts are an effective approach to boost the economy.

Keywords: debt, debt to GDP ratio, debt dynamics, government spending.
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Camilla Yanushevskyα & Rafael Yanushevskyσ

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Author α: Standard & Poor’s (S&P) Global Market Intelligence, East Market Street Charlottesville, VA 22902, USA.

σ: Research & Technology Consulting, 5106 Danbury Rd., Bethesda, MD 20814, USA.

II. INTRODUCTION

The two camps of economists have different views concerning how to improve the economy in times of economic downturn. Representatives of the first camp do not believe that a large national debt will inevitably undermine economic growth and can even throw the economy into recession (comparing current debt levels with that of a number of advanced countries, the U.S. included, have had in the past and dealt with). They consider government spending on infrastructure as an efficient strategy and support the approach based on additional government borrowing. Another group of economists that concerns with high government debt which, as they believe, can inevitably undermine economic growth, supports austerity measures. Economists belonging to this camp consider the solution of the huge national debt problem as an urgent task. They believe that the approach based on additional government borrowing with a hope that this will help decrease the debt in the future has less probability of success than immediate austerity measures.

The literature on the relationship between government debt and economic growth is scarce. However, the findings of both Herndon et al. (2013) and Reinhart and Rogoff (2010) are suggestive, rather than conclusive, since they operate with past historic data. It is dangerous to build future financial policy by using blindly such findings since pictures of the world economy are changing with time, and statistics of the past may not apply to a current or future economic situation in a country. More reliable mathematical models should be developed.

Research results related to the debt to GDP ratio were based mostly on analysis of the existing statistical economic data. Different conclusions and following disputes reflect different interpretation by economists of the available statistical material. Using regression models and/or the existing historic data most of the related publications examined the impact of government spending and tax cuts to stimulate the economy on GDP or analyzed the influence of the debt to GDP ratio on economic growth. However, they did not establish the direct relationship between GDP, the related government spending and/or tax cuts and the debt to GDP ratio.

Below models describing the debt to GDP ratio dynamics are considered to examine the linkage...
between the GDP growth rate, the related government spending, maintained by government borrowing, as well as tax cuts (their effect is presented by related fiscal multipliers) and the debt to GDP ratio. The developed models in Yanushevsky (2013) and Yanushevsky (2014) analyze the impact of expansionary fiscal policy, intended to reduce unemployment and increase economic growth by using stimulus packages, based on the lower estimate of the debt to GDP ratio. The models analyze whether government stimulus spending can improve the economic situation or this policy increases the national debt to such a degree that the debt to GDP ratio becomes dangerously high.

In Yanushevsky (2013) and Yanushevsky (2014) main attention is paid to the effect of government investment in infrastructure. The considered models estimating the lower debt to GDP ratio limit that can be achieved by the government stimuli are described by differential and difference equations with a constant coefficients. The main components of these models, fiscal multipliers, do not change in time and start acting immediately. In reality, a real effect of stimuli becomes visible at least about a year later. As a rule, stimulus is injected into the economy by steps; its implementation is distributed in time. The latest studies (see, e.g. Christiano et al., 2011) show that the stimulus impact is more substantial when fiscal policy is accompanied with a proper monetary policy. In this case, the fiscal multiplier is higher at the initial period, when it start working, and then, with the GDP rate growth, its impact decreases significantly.

The earlier used assumption that stimuli change the GDP growth from \( g_0 \) to \( g_1 \) enables us to evaluate roughly the efficiency of the considered stimulus policies for high debt to GDP ratios. However, a sharp change of a growth rate at \( t = t_0 \) is a rather strong assumption to obtain the debt to GDP ratio estimate very close to reality. The reaction to stimuli is more complicated. The GDP growth rate is not constant and it is influenced by many other factors, not included in the models, that require special government actions reflected in its spending and revenues.

In this paper, a generalized debt to GDP ratio dynamic model is developed to obtain more accurate estimates of the debt to GDP ratio, especially for the case when the considered fiscal stimuli are accompanied by a proper monetary policy. Below we consider the debt to GDP ratio dynamic model with time-varying parameters, which estimate more precisely the debt to GDP ratio than the examined earlier models (see Yanushevsky, 2013; Yanushevsky, 2014).

### III. GENERALIZED DEBT TO GDP RATIO MODEL

The debt to GDP ratio dynamics \( d(t) \) can be presented as

\[
\dot{d}(t) = ( r + g) d(t) + \frac{G(t) - T(t)}{Y(t)} \tag{2.1}
\]

where \( Y \) is the GDP; \( g \) is the GDP growth rate; \( r \) is the interest rate on debt; \( G(t) \) and \( T(t) \) are government purchases (expenditure excluding interest payments on debt) and revenues, respectively.

Government spending \( G(t) \) and revenues \( T(t) \) are presented in the form

\[
G(t) = l_i(t)Y(t) \tag{2.2}
\]

\[
T(t) = \tau(t)Y(t)
\]

where \( \tau(t) \) is a tax rate and \( l_i(t) \) reflects the level of government spending.

In contrast to the above mentioned models we assume that the government spending \( G(t) \) not limited by \( G(t_0) \) (at the moment of stimuli actions), so that \( \tau(t) \) and \( l(t) \) are functions of time. It means that the model

\[
\dot{d}(t) = (r(t) - g_0(t))d(t) + l(t) - \tau(t) \tag{2.3}
\]

evaluates \( d(t) \) on a finite interval \( t \in [t_0, t_f] \) under
a certain economic policy (we analyze the policy with \( G(t) \) and \( T(t) \) that created an economic crises with the GDP growth rate \( g_0(t) \) and a high unemployment), and the problem is to evaluate the efficiency of stimuli measures that would produce the GDP growth \( g(t) > g_0(t) \) and decrease unemployment.

For the time-varying GDP growth rate we have

\[
Y(t) = Y(t_0)e^{\int_{t_0}^{t} g(d)dt}, \quad t \geq t_0
\]  
(2.4)

The additional government spending \( \Delta G(t) \) at \( t \geq t_0 \) that makes in the short run the GDP growth rate \( g(t) \) can be presented as

\[
\Delta G(t) = I_G^{-1}(t)(e^{\int_{t_0}^{t} g(d)dt} - e^{0})Y(t_0),
\]  
(2.5)

(Here \( \tau(t) \) corresponds to the old tax policy, where

\[
I_G(t) = (p_1 I_G^{-1}(t) + I_T^{-1} p_2(t))^{-1}
\]  
(2.8)

where the coefficients \( p_1 + p_2 = 1 \) reflect the share of each considered measure presented by the corresponding multiplier (see (2.5) and (2.6)).

Similar to Yanushevsky (2013) the decrease of government spending by \( \Delta G_1(t) \) due to the increase in employment can be evaluated approximately as

\[
\Delta G_1(t) = c l_2 G(t_0)(e^{\int_{t_0}^{t} g_1^0(t)dt} - 1),
\]  
(2.9)

where \( I_G(t) \) is the spending multiplier (the above equation follows directly from the definition of the spending multiplier).

The additional tax cut \( \Delta \tau(t) \) at \( t \geq t_0 \) that increases \( Y(t) \) and makes it growing with the rate \( g(t) \) (i.e., \( Y(t) = Y(t_0)e^{\int_{t_0}^{t} g(d)dt} \)) in the short run will decrease the tax revenues by

\[
\Delta T(t) = I_T^{-1}(t)(e^{\int_{t_0}^{t} g(d)dt} - e^{0})Y(t_0),
\]  
(2.6)

where \( I_T(t) \) is the tax multiplier (the above equation follows directly from the definition of the tax multiplier).

As a result, the new government revenues \( T(t) \) will be

\[
T(t) = \tau(t)e^{\int_{t_0}^{t} g(d)dt} Y(t_0) - I_T^{-1}(t)(e^{\int_{t_0}^{t} g(d)dt} - e^{0})Y(t_0)
\]  
(2.7)

where \( l_2 \) characterizes the percent of welfare related spending at \( t = t_0 \); the values of \( c > 0 \) reflect a country's welfare policy and structure; \( \beta \) is the output elasticity of labor of the Cobb-Douglas function (see, e.g., Felipe and Adams, 2005) used as the estimation and forecasting of GDP from the supply side (the weight of (2.9) is significantly smaller than (2.5) and (2.6)).

The generalized debt to GDP ratio dynamics is described by the following equation obtained by substituting (2.2)-(2.9) in (2.1)
where \( G(t) \) is the unrelated to stimuli part of spending and, in the case of tax cuts \( \tau(t) \) is not a real tax to GDP ratio; it corresponds to the old tax policy \( T_{old}(t) \).

The above equation can be transformed to

\[
\dot{d}(t) = (r(t) - g(t))d(t) + l_1(t) - c\int_0^t l_1(t) e^{-\tau(t)} dt + \int_0^t g(t)e^{-\tau(t)} dt, \quad t > t_0
\]

(If only stimulus spending is considered, the last term of (2.11) is equal to \( \tau(t) \).)

To analyze the efficiency of the policy combining fiscal stimuli with decreasing other government spending we assume that government projected earlier spending \( G(t) \) not related to the considered stimuli decreases with a rate \( h \), that is, in (2.10) it should be \( G(t)e^{-h(t-t_0)} \) instead of \( G(t) \).

Formally, as mentioned earlier, the \( l_1(t) \) values reflect the level of government spending, so that if the forecasted data about future government spending is close to reality the model (2.11) can be used also to estimate the effect of decreased government spending if spending cuts are expected in the future.

The solution of the above equations (2.11) and (2.12) enables one to analyze the efficiency of fiscal and monetary policies focused to improve a country’s economy. However, the reliability of the obtained results depends upon the properly chosen multipliers and the accuracy of the forecasting parameters of the model.

For practical purposes the discrete analog of the above equations is more preferable. The following equivalents of the continuous model (2.4)-(2.9) should be included in the discrete model (the below notations are obvious; see also

\[
Y_{t+1} = (1 + g_{t+1})Y_t, \quad t = 0, 1, 2, \ldots \tag{2.13}
\]

where \( g_{t+1} \) is the GDP growth rate in the \((t + 1)\) year.

The multiplier’s effect (2.5) and (2.6) can be presented by

\[
\Delta G_{t+1} = \int_0^{t+1} \left[ \prod_{i=1}^{t+1} (g_i + 1) - \prod_{i=1}^{t+1} (g_{0i} + 1) \right] Y_0, \tag{2.14}
\]

and

\[
\Delta T_{t+1} = \int_0^{t+1} \left[ \prod_{i=1}^{t+1} (g_i + 1) - \prod_{i=1}^{t+1} (g_{0i} + 1) \right] Y_0 \tag{2.15}
\]

Instead of (2.9) we have

\[
\Delta G_{t+1} = cl_2 G_0 \left( \int_0^{t+1} \prod_{i=1}^{t+1} (g_i + 1)(1 + g_{0i}) \right) - 1 \tag{2.16}
\]
The discrete analog of the model (2.10) has the following form:

$$d_{t+1} = \frac{1+\tau_{r,1}}{1+g_{r,1}} d_t - \frac{\tau_{r,1}}{\prod_{i=1}^{\mu}(g_{i+1})} \left( \sum_{i=1}^{\mu} \frac{1}{\prod_{j=1}^{\nu}(g_{j+1})} \right) + \frac{G_{r,1} - c_t \sum_{i=1}^{\mu} (\prod_{j=1}^{\nu}(g_{j+1}))^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} + \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1}}{\prod_{i=1}^{\mu}(g_{i+1})} \right) Y_{t-1}$$

(2.17)

$$t = 0, 1, 2, \ldots$$

or since $G_t = l_{t,1}$, the above equation can be transformed to

$$d_{t+1} = \frac{1+\tau_{r,1}}{1+g_{r,1}} d_t + l_{t,1} - \frac{\tau_{r,1}}{\prod_{i=1}^{\mu}(g_{i+1})} \left( \sum_{i=1}^{\mu} \frac{1}{\prod_{j=1}^{\nu}(g_{j+1})} \right) + \frac{c_t \sum_{i=1}^{\mu} (\prod_{j=1}^{\nu}(g_{j+1}))^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} + \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1}}{\prod_{i=1}^{\mu}(g_{i+1})} \right) Y_{t-1}$$

(2.18)

$$t = 0, 1, 2, \ldots$$

where for the case of tax cuts, similar to the continuous model (see (2.10)), $\tau_{r,1}$ does not present the real tax to GDP ratio; in the case of only spending stimulus the tax component of the above equations is equal to $\tau_{r,1}$.

The efficiency of the government fiscal policy combining the considered stimuli with decreasing with a rate $h$ other government spending (see (2.12)) is described by the following equation

$$d_{t+1} = \frac{1+\tau_{r,1}}{1+g_{r,1}} d_t + (1-h) l_{t,1} - \frac{\tau_{r,1}}{\prod_{i=1}^{\mu}(g_{i+1})} \left( \sum_{i=1}^{\mu} \frac{1}{\prod_{j=1}^{\nu}(g_{j+1})} \right) + \frac{c_t \sum_{i=1}^{\mu} (\prod_{j=1}^{\nu}(g_{j+1}))^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} + \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1} \prod_{j=1}^{\nu}(g_{j+1})^{-1}}{\prod_{i=1}^{\mu}(g_{i+1})} \right) Y_{t-1}$$

(2.19)

$$t = 0, 1, 2, \ldots$$

As mentioned in the case of the continuous model (2.11), the $l_{t,1}$ values ($t = 0, 1, 2, \ldots$) reflect the level of government spending, so that formally (2.19) can be used also to describe the effect of decreased government spending (in the case of only spending stimulus the tax component of the above equations is equal to $\tau_{r,1}$).

The expressions (2.18) and (2.19) are written similar to (2.10) and (2.11). However, taking into account that the stimulus measures are implemented in steps and distributed over a certain time interval, it is reasonable to consider the following relation between the $g_{0,i}$ and $g_i$ : $g_{0,i} = g_{i+1}$ ($i = 2, 3, \ldots$), which means that the system dynamics on the $i$-th time interval starts from the projected growth rate for the previous interval. In this case, (2.18) and (2.19) can be presented in the form.
As mentioned earlier, the combination of government fiscal and monetary policy is more preferable since in the case of many uncertain parameters simpler models can produce better results. Yanushevsky (2013) and Yanushevsky (2014) are predicted properly. Otherwise, the models of countries more dynamic and with less bureaucracy than leading industrial countries. Of course, other possible scenarios for can be considered that would simplify (2.18) and (2.19).

The considered debt to GDP ratio dynamic models with time-varying parameters can be used successfully only if the time-varying coefficients are predicted properly. Otherwise, the models of Yanushevsky (2013) and Yanushevsky (2014) are preferable since in the case of many uncertain parameters simpler models can produce better results.

IV. SIMULATION RESULTS

As mentioned earlier, the combination of government fiscal and monetary policy is more effective than a “pure” fiscal policy and the fiscal multiplier changes significantly during the recovery process (see, e.g., Christiano et al., 2011). Here we use the model (2.18) to examine the debt to GDP trajectory for this case.

We consider here the effect of an increase in government spending for monetary policy near zero and at the zero lower bound. This increase leads to a rise in output and expected inflation. With the nominal interest rate near zero, the rise in expected inflation drives down the real interest rate, which drives up private spending. This rise in spending leads to a further rise in output and expected inflation and a further decline in the real interest rate. The net result is a large rise in output and a large fall in the rate of deflation. In effect, the increase in government consumption counteracts the deflationary spiral associated with the zero-bound state. An important practical objection to using fiscal policy to counteract a contraction associated with the zero-bound state is that there are long lags in implementing increases in government spending. Motivated by this consideration, Christiano et al. (2011) studied the size of the government-spending multiplier in the presence of implementation lags. As indicated earlier, when the nominal interest rate is zero or near zero, then there is a large effect on current output and the fiscal multiplier is about 3.8. However, in future periods, in which the nominal interest rate is positive and higher, the effect on government spending is smaller, and the government-spending multiplier becomes close to 1.

We analyze the debt to GDP ratio change from 2009 to 2015 based on the model (2.18) with the GDP growth rate \( g_0 = -0.014 \), \( d_0 = 0.7354 \), \( l_0 = 0.18 \), and \( l_2 = 0.11 \) that correspond to the U.S. government data related to the fourth quarter of 2008 and the first two quarters of 2009. Since

\[
d_{t+1} = \frac{1+r_{t+1}}{\sum_{i=1}^{10} l_i} d_t + l_{1,t+1} - \tau_{t+1} \frac{g_{01}+1}{g_{01}+1}
\]

\[
d_{t+1} = \frac{1+r_{t+1}}{\sum_{i=1}^{10} l_i} d_t + (1-h)^{t+1} l_{1,t+1} - \tau_{t+1} \frac{g_{01}+1}{g_{01}+1}
\]

\[
d_{t+1} = \frac{1+r_{t+1}}{\sum_{i=1}^{10} l_i} d_t + (1-h)^{t+1} l_{1,t+1} - \tau_{t+1} \frac{g_{01}+1}{g_{01}+1}
\]
Table 2.1: Simulation results for the debt to GDP ratio dynamics model (2.18)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$g_0$%</td>
<td>0.34</td>
<td>2.532</td>
<td>1.6</td>
<td>2.223</td>
<td>1.678</td>
<td>2.37</td>
<td>2.6</td>
</tr>
<tr>
<td>$\tau_{t+1}$</td>
<td>0.15</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.175</td>
<td>0.175</td>
<td>0.185</td>
</tr>
<tr>
<td>$l_{1,t+1}$</td>
<td>0.182</td>
<td>0.144</td>
<td>0.15</td>
<td>0.155</td>
<td>0.141</td>
<td>0.14</td>
<td>0.138</td>
</tr>
<tr>
<td>$r$%</td>
<td>3.35</td>
<td>3.05</td>
<td>2.89</td>
<td>2.588</td>
<td>2.43</td>
<td>2.4</td>
<td>2.35</td>
</tr>
<tr>
<td>$l_3$</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>3.5</td>
<td>3.8</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Debt/GDP ratio $d$</td>
<td>(0.8236)</td>
<td>(0.904)</td>
<td>(0.9515)</td>
<td>(0.9935)</td>
<td>(1.003)</td>
<td>(1.03)</td>
<td>(1.038)</td>
</tr>
<tr>
<td>Debt/GDP ratio $d$</td>
<td>0.8262</td>
<td>0.8766</td>
<td>0.9299</td>
<td>0.976</td>
<td>1.0043</td>
<td>1.032</td>
<td>1.061</td>
</tr>
</tbody>
</table>

Table 2.2: Simulation results for the debt to GDP ratio dynamics model (2.19)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Debt/GDP ratio $d$</td>
<td>0.8190</td>
<td>0.8609</td>
<td>0.9023</td>
<td>0.9334</td>
<td>0.9454</td>
<td>0.9542</td>
<td>0.963</td>
</tr>
</tbody>
</table>

The simulation results are very close the real U.S. government data (see Table 2.1, the debt to GDP ratio values in parenthesis). The errors are, to some extent, the result of insufficient and reliable information concerning the U.S. stimulus available now. According to many economists the $800 billion American Recovery and Reinvestment Act does not present the real amount of money pumped into the economy during 2008-2015, which is much higher. According to the Cato Institute estimates the U.S. has dumped at least $2.5 trillion of fiscal stimulus into the economy since 2008. Art Laffer, economist and former economics adviser to President Ronald Reagan, also puts the true amount of stimulus spending much higher than $800 billion — at more than $4 trillion. The health care costs related to the President’s care program jumped significantly in 2014 and 2015, the government financing of solar energy incentive programs is also increased in 2015. These and other factors (e.g., government constantly corrects economic data; the economic data provided by different respectful organizations may differ significantly) make it difficult to use very efficiently the generalized
model. However, the presented simulations reached the main goal and showed that the financial multiplier under the coordinated fiscal and monetary policy can be high, as it was indicated by Christiano et al. (2011). The effect of the 2008-2015 tax cuts is significantly lower than the stimulus spending. That is why \( l_{t+1,1} \) should be close to \( l_{G,t+1} \) and we tested \( l_{t+1,1} \geq 3.5, t = 4-7 \) and \( l_{t+1,1} = 0.5 \) since the 2009 stimulus formally started acting only in the last two quarters of 2009.

As seen from the previous material, many economists believe (their research supports their opinion) that in the period of financial crises financial multipliers are low, around 1. Different models are built to prove that (see e.g., Auerbach and Gorodnichenko (2012); Blanchard and Perotti (2002); Heim (2012); Ilzetzki et al. (2013); Mankiw (2008)). The considered generalized model, totally different from the mentioned models, can be used to determine the multipliers \( l_{G,t+1} \) or \( l_{t+1} \) if in (2.18) [(2.11)] we consider \( l_{G,t} \) as a function that minimizes, for example, \[ \sum_{t=1}^{n} (d_t - d_0)^2 \] subject to \( const1 < l_{G,t} < const2 \),

where \( d_t \) (\( t = 1, \ldots, n \)) are known values of the debt to GDP ratio (in the above example this is the 2009-2015 data). Since this problem belongs to the class of inverse problems and bears all difficulties of solving such problems (e.g., unrobustness of the solution, its high sensitivity to the accuracy of used data, and the need of regularization used for solving ill-posed problems).

\section*{V. CONCLUSION}

The developed generalized debt to GDP ratio dynamics model analyzes whether government stimulus spending and/or tax cuts can improve the economic situation and whether this government fiscal policy accompanied with a proper monetary policy can move the economy on a more productive stage. Analysis of the developed debt to GDP ratio dynamics model for the data characterizing the state of the U.S. economy during the 2009-2015 period shows that the government stimulus policy alone cannot decrease the debt to GDP ratio. In order to adequately fund public infrastructure, innovative new funding mechanisms should be found that do not burden rising deficits and likely would stimulate the private sector (for example, using public-private partnerships). Only investment in the areas which would bring a substantial profit and growth of capital, as well as austerity measures can increase economic growth and decrease the debt to GDP ratio.

\section*{REFERENCES}


